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Unit 4
Algebraic Expressions and Algebraic Formulas

EXERCISE 4.1

Polynomials

A polynomial in the variable x is an algebraic expression of the form

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots + a_1 x + a_0, \quad a_n \neq 0$$

Where n, the highest power of x, is a non-negative integer called the *degree* of the polynomial and each coefficient a_n is a real number.

Q1. Identify whether the following algebraic expressions are polynomials (yes or not).

(i) $3x^2 + \frac{1}{x} - 5$ **(ii)** $3x^3 - 4x^2 - x\sqrt{x} + 3$

(iii) $x^2 - 3x + \sqrt{2}$ **(iv)** $\frac{3x}{2x-1} + 8$

Solution:

(i) No	(ii) No	(iii) Yes	(iv) No

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Q2. State whether each of the following expression is a rational expression or not.

(i) $\frac{3\sqrt{x}}{3\sqrt{x}+5}$ **(ii)** $\frac{x^3 - 2x^2 + \sqrt{3}}{2+3x-x^2}$

(ii) $\frac{x^2+6x+9}{x^2-9}$ **(iv)** $\frac{2\sqrt{x}+3}{2\sqrt{x}-3}$

Solution:

(i) No	(ii) Yes	(iii) Yes	(iv) No

Q3. Reduce the following rational expressions to the lowest forms.

(i) $\frac{120\,x^2y^3z^5}{30\,x^3y\,z^2}$ **(ii)** $\frac{8a(x+1)}{2(x^2-1)}$

(iii) $\frac{(x+y)^2 - 4xy}{(x-y)^2}$ **(iv)** $\frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$

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(v) $\dfrac{(x+1)(x^2-1)}{(x+1)(x^2-4)}$ **(vi)** $\dfrac{x^2-4x+4}{2x^2-8}$

(vii) $\dfrac{64x^5-64x}{(8x^2+8)(2x+2)}$ **(viii)** $\dfrac{9x^2-(x^2-4)^2}{4+3x-x^2}$

Solution:

(i) $\dfrac{120\,x^2y^3z^5}{30\,x^3y\,z^2} = \dfrac{30\times 4y^{3-1}\times z^{5-2}}{30\,x^{3-2}} = \dfrac{4\,y^2z^3}{x}$

(ii) $\dfrac{8a(x+1)}{2(x^2-1)} = \dfrac{2(4a)(x+1)}{2(x+1)(x-1)} = \dfrac{4a}{x-1}$

(iii) $\dfrac{(x+y)^2-4xy}{(x-y)^2} = \dfrac{x^2+y^2+2xy-4xy}{(x-y)^2}$

$$= \frac{x^2+y^2-2xy}{(x-y)^2} = \frac{(x-y)^2}{(x-y)^2} = 1$$

(iv) $\dfrac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)} = \dfrac{(x^3-y^3)(x-y)^2}{(x-y)(x^2+xy+y^2)}$

$$= \frac{(x-y)(x^2+xy+y^2)(x-y)^2}{(x-y)(x^2+xy+y^2)}$$

$$= (x-y)^2$$

(v) $\dfrac{(x+1)(x^2-1)}{(x+1)(x^2-4)} = \dfrac{(x+2)(x+1)(x-1)}{(x+1)(x+2)(x-1)} = \dfrac{x-1}{x-2}$

(vi) $\dfrac{x^2-4x+4}{2x^2-8} = \dfrac{x^2-4x+2^2}{2(x^2-4)} = \dfrac{(x-2)^2}{2(x+2)(x-2)} = \dfrac{x-2}{2(x+2)}$

(vii) $\dfrac{64x^5-64x}{(8x^2+8)(2x+2)} = \dfrac{64x\,(x^4-1)}{8(x^2+1)2(x+1)}$

$$= \frac{64x(x^2+1)(x^2-1)}{16(x^2+1)(x+1)}$$

$$= \frac{4x(x^2-1)}{x+1} = \frac{4x(x+1)(x-1)}{x+1}$$

$$= 4x\,(x-1)$$

(viii) $\dfrac{9x^2-(x^2-4)^2}{4+3x-x^2} = \dfrac{(3x)^2-(x^2-4)^2}{4+3x-x^2}$

$$= \frac{[3x+(x^2-4)][3x-(x^2-4)]}{4+3x-x^2}$$

$$= \frac{(3x+x^2-4)(3x-x^2+4)}{4+3x-x^2}$$

$$= \frac{(x^2+3x-4)(4+3x-x^2)}{4+3x-x^2} = x^2+3x-4$$

Q4. **Evaluate (a)** $\dfrac{x^3y-2z}{xz}$ **for**

(i) $x=3,\ y=-1,\ z=-2$

(ii) $x=-1,\ y=-9,\ z=4$

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(b) $\dfrac{x^2y^3-5z^4}{xyz}$ **for** $x=4,\ y=-2,\ z=-1$

Solution:

(a) $\dfrac{x^3y-2z}{xz}$

(i) Putting $x=3, y=-1, z=-2$

$$\frac{x^3y-2z}{xz}=\frac{(\ )^3(-1)-2(-2)}{3(-2)}$$

$$=\frac{-27+4}{-6}=\frac{-23}{-6}=\frac{23}{6}=3\frac{5}{6}$$

(ii) Putting $x=-1, y=-9, z=4$

$$\frac{x^3y-2z}{xz}=\frac{(-1)^3(-9)-2(4)}{(-1)(4)}$$

$$=\frac{9-8}{-4}=\frac{1}{-4}=-\frac{1}{4}$$

(b) $\dfrac{x^2y^3-5z^4}{xyz}$

Putting $x=4,\ y=-2, z=-1$

$$\frac{x^2y^3-5z^4}{xyz}$$

$$=\frac{(4)^2(-2)^3-5(-1)^4}{4(-2)(-1)}$$

$$=\frac{16(-8)-5(1)}{8}=\frac{-128-5}{8}=-\frac{133}{8}=-16\frac{5}{8}$$

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Q5. Perform the indicated operation and simplify.

(i) $\dfrac{15}{2x-3y}-\dfrac{4}{3y-2x}$ **(ii)** $\dfrac{1+2x}{1-2x}-\dfrac{1-2x}{1+2x}$

(iii) $\dfrac{x^2-25}{x^2-36}-\dfrac{x+5}{x+6}$ **(iv)** $\dfrac{x}{x-y}-\dfrac{y}{x+y}-\dfrac{2xy}{x^2-y^2}$

(v) $\dfrac{x-2}{x^2+6x+9}-\dfrac{x+2}{2x^2-18}$

(vi) $\dfrac{1}{x-1}-\dfrac{1}{x+1}-\dfrac{2}{x^2+1}-\dfrac{4}{x^4-1}$

Solution:

(i) $\dfrac{15}{2x-3y}-\dfrac{4}{3y-2x}$

$$=\frac{15}{2x-3y}-\frac{4}{-2x+3y}=\frac{15}{2x-3y}-\frac{4}{-(2x-3y)}$$

$$=\frac{15}{2x-3y}+\frac{4}{2x-3y}=\frac{15+4}{2x-3y}=\frac{19}{2x-3y}$$

(ii) $\dfrac{1+2x}{1-2x}-\dfrac{1-2x}{1+2x}$

$$=\frac{(1+2x)^2-(1-2x)^2}{(1-2x)(1+2x)}$$

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$$= \frac{1+4x+4x^2-(1-4x+4x^2)}{(1-2x)(1+2x)}$$

$$= \frac{1+4x+4x^2-1+4x-4x^2}{(1-2x)(1+2x)} = \frac{8x}{1-4x^2}$$

(iii) $\dfrac{x^2-25}{x^2-36} - \dfrac{x+5}{x+6}$

$$= \frac{(x+5)(x-5)}{(x+6)(x-6)} - \frac{x+5}{x+6}$$

$$= \frac{x+5}{x+6}\left(\frac{x-5}{x-6} - 1\right)$$

$$= \frac{x+5}{x+6}\left\{\frac{x-5-(x-6)}{x-6}\right\}$$

$$= \frac{x+5}{x+6}\left(\frac{x-5-x+6}{x-6}\right)$$

$$= \frac{x+5}{x+6}\cdot\frac{1}{x-6} = \frac{x+5}{x^2-36}$$

(iv) $\dfrac{x}{x-y} - \dfrac{y}{x+y} - \dfrac{2xy}{x^2-y^2}$

$$= \frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{(x+y)(x-y)}$$

$$= \frac{x(x+y)-y(x-y)-2xy}{(x+y)(x-y)}$$

$$= \frac{x^2+xy-xy+y^2-2xy}{(x+y)(x-y)}$$

$$= \frac{x^2+y^2-2xy}{(x+y)(x-y)} = \frac{(x-y)^2}{(x+y)(x-y)} = \frac{x-y}{x+y}$$

(v) $\dfrac{x-2}{x^2+6x+9} - \dfrac{x+2}{2x^2-18}$

$$= \frac{x-2}{(x+3)^2} - \frac{x+2}{2(x^2-9)}$$

$$= \frac{x-2}{(x+3)^2} - \frac{x+2}{2(x^2-9)}$$

$$= \frac{x-2}{(x+3)^2} - \frac{x+2}{2(x+3)(x-3)}$$

$$= \frac{(x-2)2(x-3)-(x+2)(x+3)}{2(x+3)^2(x-3)}$$

$$= \frac{2(x^2-5x+6)-(x^2+5x+6)}{2(x+3)^2(x-3)}$$

$$= \frac{2x^2-10x+12-x^2-5x-6}{2(x+3)^2(x-3)}$$

$$= \frac{x^2-15x+6}{2(x+3)^2(x-3)}$$

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(vi) $\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$

$= \frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{(x^2+1)(x+1)(x-1)}$

$= \frac{(x^2+1)(x+1) - (x^2+1)(x-1) - 2(x+1)(x-1) - 4}{(x^2+1)(x+1)(x-1)}$

$= \frac{x^3+x^2+x+1 - (x^3 - x^2 + x - 1) - 2(x^2-1) - 4}{x^4-1}$

$= \frac{x^3+x^2+x+1-x^3+x^2-x+1-2x^2+2-4}{x^4-1}$

$= \frac{0}{x^4-1} = 0$

Q6. Perform the indicated operation and simplify.

(i) $(x^2-49).\frac{5x+2}{x+7}$ **(ii)** $\frac{4x-12}{x^2-9} \div \frac{18-2x^2}{x^2+6x+9}$

(iii) $\frac{x^6-y^6}{x^2-y^2} \div (x^4+x^2y^2+y^4)$

(iv) $\frac{x^2-1}{x^2+2x+1} . \frac{x+5}{1-x}$

(v) $\frac{x^2+xy}{y(x+y)} . \frac{x^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y}$

Solution:

(i) $(x^2-49).\frac{5x+2}{x+7}$

$= \frac{(x+7)(x-7)(5x+2)}{x+7}$

$= (x-7)(5x+2)$

$= 5x^2+2x-35x-14$

$= 5x^2-33x-14$

(ii) $\frac{4x-12}{x^2-9} \div \frac{18-2x^2}{x^2+6x+9}$

$= \frac{4x-12}{x^2-9} \times \frac{x^2+6x+9}{18-2x^2}$

$= \frac{4(x-3)}{(x+3)(x-3)} \times \frac{(x+3)^3}{2(9-x^2)}$

$= \frac{2(x-3)\times(x+3)\times(x+3)}{(x+3)(x-3)(3-x)(3+x)}$

$= \frac{-2(3-x)(3+x)(x+3)}{(x+3)(x-3)(3-x)(x+3)} = \frac{-2}{x-3} = \frac{2}{3-x}$

(iii) $\frac{x^6-y^6}{x^2-y^2} \div (x^4+x^2y^2+y^4)$

$= \frac{x^6-y^6}{x^2-y^2} \times \frac{1}{x^4+x^2y^2+y^4}$

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$$= \frac{(x^3+y^3)(x^3-y^3)}{(x+y)(x-y)} \times \frac{1}{x^4+2x^2y^2+y^4-x^2y^2}$$

$$= \frac{(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)}{(x+y)(x-y)[(x^2+y^2)^2-(xy)^2]}$$

$$= \frac{(x^2-xy+y^2)(x^2+xy+y^2)}{(x^2+xy+y^2)(x^2-xy+y^2)} = 1$$

(iv) $$\frac{x^2-1}{x^2+2x+1} \cdot \frac{x+5}{1-x}$$

$$= \frac{(x+1)(x-1)}{(x+1)^2} \cdot \frac{x+5}{1-x}$$

$$= \frac{-(x+1)(1-x)(x+5)}{(x+1)(x+1)(1-x)}$$

$$= \frac{-(x+5)}{x+1}$$

(v) $$\frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y}$$

$$= \frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} \cdot \frac{xy-2y}{x^2-x}$$

$$= \frac{x(x+y)}{y(x+y)} \cdot \frac{x(x+y)}{y(x+y)} \cdot \frac{y(x-2)}{x(x-1)}$$

$$= \frac{x(x-2)}{y(x-1)}$$

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EXERCISE 4.2

Q1. **(i)** **If $a+b=10$ and $a-b=6$, then find the value of (a^2+b^2).**

(ii) **If $a+b=5$, $a-b=\sqrt{17}$, then find the value of ab.**

Solution:

(i) $a+b=10, \quad a-b=6$

$(a+b)^2+(a-b)^2=2(a^2+b^2)$

$(10)^2+(6)^2=2(a^2+b^2)$

$100+36=2(a^2+b^2)$

$2(a^2+b^2)=136$

$a^2+b^2=68$

(ii) $a+b=5, \quad a-b=\sqrt{17}$

$(a+b)^2-(a-b)^2=4ab$

$(5)^2-(\sqrt{17})^2=4ab$

$25-17=4ab$

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or $\quad 4ab = 8$

$\Rightarrow \quad ab = 2$

Q2. If $a^2 + b^2 + c^2 = 45$ and $a + b + c = -1$, find the value of $ab + be + ca$.

Solution:

$$a^2 + b^2 + c^2 = 45, \qquad a + b + c = -1$$
$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$
$$(-1)^2 = 45 + 2(ab + bc + ca)$$
$$1 = 45 + 2(ab + bc + ca)$$
$$1 - 45 = 2(ab + bc + ca)$$

or $\quad 2(ab + bc + ca) = -44$

$\Rightarrow \quad ab + bc + ca = -22$

Q3. If $m + n + p = 10$ and $mn + np + mp = 27$, find the value of $m^2 + n^2 + p^2$.

Solution:

$$(m + n + p)^2 = m^2 + n^2 + p^2 + 2(mn + np + mp)$$
$$(10)^2 = m^2 + n^2 + p^2 + 2(27)$$
$$100 = m^2 + n^2 + p^2 + 54$$

So $\quad m^2 + n^2 + p^2 = 100 - 54 = 46$

Q4. If $x^2 + y^2 + z^2 = 78$ and $xy + yz + zx = 59$, find the value of $x + y + z$.

Solution:

$$x^2 + y^2 + z^2 = 78, xy + yz + zx = 59$$
$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$$
$$= 78 + 2(59)$$
$$(x + y + z)^2 = 78 + 118 = 196$$

$\Rightarrow \quad x + y + z = \pm\sqrt{196} = \pm 14$

Q5. If $x + y + z = 12$ and $x^2 + y^2 + z^2 = 64$, find the value of $xy + yz + zx$.

Solution:

$$x + y + z = 12, \qquad x^2 + y^2 + z^2 = 64$$
$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$$
$$(12)^2 = 64 + 2(xy + yz + zx)$$
$$144 = 64 + 2(xy + yz + zx)$$

or $\quad 2(xy + yz + zx) = 144 - 64 = 80$

$\Rightarrow \quad xy + yz + zx = 40$

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Q6. If $x + y = 7$ and $xy = 12$, then find the value of $x^3 + y^3 + z^3$.

Solution:

$x + y = 7, \quad xy = 12$

$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$(7)^3 = x^3 + y^3 + 3(12)(7)$

$343 = x^3 + y^3 + 252$

or $x^3 + y^3 = 343 - 252 = 91$

Q7. If $3x + 4y = 11$ and $xy = 12$, then find the value of $27x^3 + 64y^3$.

Solution:

$3x + 4y = 11, xy = 12.$

$(3x + 4y)^3 = (3x)^3 + (4y)^3 + 3(3x)(4y)(3x + 4y)$

$(11)^3 = 27x^3 + 64y^3 + 36xy(11)$

$1331 = 27x^3 + 64y^3 + 396(12)$

$1331 = 27x^3 + 64y^3 + 4752$

or $27x^3 + 64y^3 = 1331 - 4752$

$\therefore$ $27x^3 + 64y^3 = -3421$

Q8. If $x - y = 4$ and $xy = 21$, then find the value of $x^3 - y^3$.

Solution:

$x - y = 4, \quad xy = 21$

$(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(4)^3 = x^3 - y^3 - 3(21)(4)$

$64 = x^3 - y^3 - 252$

$x^3 - y^3 = 64 + 252 = 316$

Q9. If $5x - 6y = 13$ and $xy = 6$, then find the value of $125x^3 - 216y^3$.

Solution:

$5x - 6y = 13, xy = 6$

$(5x - 6y)^3 = (5x)^3 - (6y)^3 - 3(5x)(6y)(5x - 6y)$

$(13)^3 = 125x^3 - 216y^3 - 90xy(13)$

$2197 = 125x^3 - 216y^3 - 1170(6)$

$2197 = 125x^3 - 216y^3 - 7020$

$125x^3 - 216y^3 = 2197 + 7020 = 9217$

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Q10. **If $x + \frac{1}{x} = 3$, then find the value of $x^3 + \frac{1}{x^3}$.**

Solution:

$$x + \frac{1}{x} = 3$$

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right)$$

$$(3)^3 = x^3 + \frac{1}{x^3} + 3(3)$$

$$27 = x^3 + \frac{1}{x^3} + 9$$

$$x^3 + \frac{1}{x^3} = 27 - 9 = 18$$

Q11. **If $x - \frac{1}{x} = 7$, then find the value of $x^3 - \frac{1}{x^3}$.**

Solution:

$$x - \frac{1}{x} = 7$$

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$(7)^3 = x^3 - \frac{1}{x^3} - 21$$

$$343 = x^3 - \frac{1}{x^3} - 21$$

or $\quad x^3 - \frac{1}{x^3} = 343 + 21 = 364$

Q12. **If $\left(3x + \frac{1}{3x}\right)$ then find the value of $\left(27x^3 - \frac{1}{27x^3}\right)$**

Solution:

$$\left(3x + \frac{1}{3x}\right)^3 = (3x)^3 + \frac{1}{(3x)^3} + 3(3x)\left(\frac{1}{3x}\right)\left(3x + \frac{1}{3x}\right)$$

$$(5)^3 = 27x^3 + \frac{1}{27x^3} + 3(5)$$

$$125 = 27x^3 + \frac{1}{27x^3} + 15$$

So $\quad 27x^3 + \frac{1}{27x^3} = 125 - 15 = 110$

Q13. **If $\left(5x - \frac{1}{5x}\right) = 6$, then find the value of $\left(125x^3 - \frac{1}{125x^3}\right)$**

Solution:

$$\left(5x - \frac{1}{5x}\right)^3 = (5x)^3 - \left(\frac{1}{5x}\right)^3 - 3(5x)\left(\frac{1}{5x}\right)\left(5x - \frac{1}{5x}\right)$$

$$(6)^3 = 125x^3 - \frac{1}{125x^3} - 3(6)$$

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$$216 = 125x^3 + \frac{1}{125x^3} - 18$$

So $\quad 125x^3 + \frac{1}{125x^3} = 216 + 18 = 234$

Q14. Factorize **(i)** $x^3 - y^3 - x + y$ **(ii)** $8x^3 - \frac{1}{27y^3}$

Solution:

(i) $x^3 - y^3 - x + y$

$= (x - y)(x^2 + xy + y^2) - 1(x - y)$

$= (x - y)(x^2 + y^2 - 1)$

(ii) $8x^3 - \frac{1}{27y^3}$

$= (2x)^3 - \left(\frac{1}{3y}\right)^3$

$= \left(2x - \frac{1}{3y}\right)\left[(2x)^2 + 2x \cdot \frac{1}{3y} + \left(\frac{1}{3y}\right)^2\right]$

$= \left(2x - \frac{1}{3y}\right)\left[4x^2 + \frac{2x}{3y} + \left(\frac{1}{3y}\right)^2\right]$

$= \left(2x - \frac{1}{3y}\right)\left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right)$

Q15. Find the products, using formulas.

(i) $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$.

(ii) $(x^3 - y^3)(x^6 + x^3y^3 + y^6)$

(iii) $(x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$

(iv) $(2x^2 - 1)(2x^2 + 1)(4x^4 + 2x^2 + 1)(4x^4 - 2x^2 + 1)$
$(4x^4 - 2x^2 + 1)(4x^4 - 2x^2 + 1)$

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Solution:

(i) $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$

$= [x^2 + y^2][(x^2)^2 - x^2 \cdot y^2 + (y^2)^2]$

$= (x^2)^3 + (y^2)^3 = x^6 + y^6$

(ii) $(x^3 - y^3)(x^6 + x^3y^3 + y^6)$

$= [x^3 - y^3][(x^3)^2 + x^3 \cdot y^3 + (y^3)^2]$

$= (x^3)^3 - (y^3)^3 = x^9 - y^9$

(iii) $(x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)(x^2 - xy + y^2)$
$(x^4 - x^2y^2 + y^4)$

$= [(x - y)(x^2 + xy + y^2)][(x + y)(x^2 - xy + y^2)]$
$[(x^2 + y^2)(x^4 - x^2y^2 + y^4)]$

$= (x^3 - y^3)(x^3 + y^3)[(x^2)^3 + (y^2)^3]$

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$= [(x^3)^2 - (y^3)^2](x^6 + y^6)$

$= [x^6 - y^6](x^6 + y^6)$

$= (x^6)^2 - (y^6)^2$

$= x^{12} - y^{12}$

(iv) $(2x^2 - 1)(2x^2 + 1)(4x^4 + 2x^2 + 1)$

$= [(2x^2 - 1)(4x^4 + 2x^2 + 1)][(2x^2 + 1)(4x^4 - 2 - 1)]$

$= (2x^2 - 1)[(2x^2)^2 + 2x^2 .1 + (1)^2][(2x^2 + 1)$
$(4x^4 - 2x^2 + 1)]$

$= (2x^2 - 1)[(2x^2)^2 + 2x^2 .1 + (1)^2](2x^2 + 1)(2x^2 + 1)^2$
$[(2x^2)^2 - (2x^2)(1) + (1)^2]$

$= [((2x^2)^3 - (1)^3)][(2x^2)^3 + (1)^3]$

$= (8x^6 - 1)(8x^6 + 1)$

$= (8x^6)^2 - (1)^2$

$= 64x^{12} - 1$

EXERCISE 4.3

Q1. Express each of the following surds in the simplest form.

(i) $\sqrt{180}$ **(ii)** $3\sqrt{162}$

(iii) $\frac{3}{4}\sqrt[3]{128}$ **(iv)** $\sqrt[5]{96x^6y^7z^8}$

Solution:

(i) $\sqrt{180} = \sqrt{90 \times 2} = \sqrt{9 \times 4 \times 5}$

$= 3 \times 2 \times \sqrt{5} = 6.\sqrt{5}$

(ii) $3\sqrt{162}$

$= 3\sqrt{81 \times 2}$

$= 3.9.\sqrt{2} = 27\sqrt{2}$

(iii) $\frac{3}{4}\sqrt[3]{128}$

$= \frac{3}{4}\sqrt[3]{64 \times 2}$

$= \frac{3}{4}\sqrt[3]{4^3 \times 2}$

$= \frac{3}{4}.4.\sqrt[3]{2} = 3.\sqrt[3]{2}$

(iv) $\sqrt[5]{96x^6y^7z^8}$

$= \sqrt[5]{32.3.x^5.x.y^5.y^3.z^5.z^3}$

$= \sqrt[5]{(2xyz)^5.3xy^2z^3}$

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$= 2xyz\sqrt[5]{3xy^2z^3}$

Q2. Simplify

(i) $\dfrac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$ (ii) $\dfrac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$

(iii) $\sqrt[5]{243\,x^5\,y^{10}z^{15}}$ (iv) $\frac{4}{5}\sqrt[3]{125}$

(v) $\sqrt{21} \times \sqrt{7} \times \sqrt{3}$

Solution:

(i) $\dfrac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$

$= \dfrac{\sqrt{18}}{\sqrt{6}} = \sqrt{\dfrac{18}{6}} = \sqrt{3}$

(ii) $\dfrac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$

$= \dfrac{\sqrt{21\times 9}}{\sqrt{63}} = \dfrac{\sqrt{189}}{\sqrt{63}} = \sqrt{\dfrac{189}{63}} = \sqrt{3}$

(iii) $\sqrt[5]{243\,x^5\,y^{10}z^{15}}$

$= (3^5\,x^5\,y^{10}\,z^{15})^{\frac{1}{5}}$

$= 3x\,y^2\,z^3$

(iv) $\frac{4}{5}\sqrt[3]{125}$

$= \frac{4}{5}\sqrt[3]{5^3} = \frac{4}{5}.5 = 4$

(v) $\sqrt{21} \times \sqrt{7} \times \sqrt{3}$

$= \sqrt{21} \times \sqrt{21}$

$= (\sqrt{21})^2 = 21$

Q3. Simplify by combining similar terms.

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

(ii) $4\sqrt{12} + 5\sqrt{27} - 3\sqrt{75} + \sqrt{300}$

(iii) $\sqrt{3}(2\sqrt{3} + 3\sqrt{3})$

(iv) $2(6\sqrt{5} - 3\sqrt{5})$

Solution:

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

$= \sqrt{9\times 5} - 3\sqrt{4\times 5} + 4\sqrt{5}$

$= 3\sqrt{5} - 3 \times 2\sqrt{5} + 4\sqrt{5}$

$= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5}$

$= (3 - 6 + 4)\sqrt{5} = \sqrt{5}$

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(ii) $4\sqrt{12}+5\sqrt{27}-3\sqrt{75}+\sqrt{300}$

$=4\sqrt{4\times 3}+5\sqrt{9\times 3}-3\sqrt{25\times 3}+\sqrt{100\times 3}$

$=4\times 2\times\sqrt{3}+5\times 3\times\sqrt{3}-3\times 5\times\sqrt{3}+10\times\sqrt{3}$

$=8\sqrt{3}+15\sqrt{3}-15\sqrt{3}+10\sqrt{3}$

$=(8+15-15+10)\sqrt{3}$

$=18\sqrt{3}$

(iii) $\sqrt{3}(2\sqrt{3}+3\sqrt{3})$

$=\sqrt{3}.\sqrt{3}\,(2+3)$

$=(\sqrt{3})^2(5)=3\times 5=15$

(iv) $2(6\sqrt{5}-3\sqrt{5})$

$=2.\sqrt{5}(6-3)$

$=2.\sqrt{5}\;.\;3=6\sqrt{5}$

Q4. Simplify

(i) $(3+\sqrt{3})(3-\sqrt{3})$ **(ii)** $(\sqrt{5}+\sqrt{3})^2$

(iii) $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$ **(iv)** $\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)$

(v) $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})(x+y)(x^2+y^2)$

Solution:

(i) $(3+\sqrt{3})(3-\sqrt{3})$

$=(3)^2-(\sqrt{3})^2=9-3=6$

(ii) $(\sqrt{5}+\sqrt{3})^2$

$=(\sqrt{5})^2+(\sqrt{3})^2+2\sqrt{5}.\sqrt{3}$

$=5+3+2\sqrt{15}$

$=8+2\sqrt{15}$

(iii) $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$

$=(\sqrt{5})^2-(\sqrt{3})^2$

$=5-3=2$

(iv) $\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)$

$=(\sqrt{2})^2-\left(\frac{1}{\sqrt{3}}\right)^2$

$=2-\frac{1}{3}=\frac{6-1}{3}=\frac{5}{3}$

(v) $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})(x+y)(x^2+y^2)$

$=\left[(\sqrt{x})^2-(\sqrt{y})^2\right](x+y)(x^2+y^2)$

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$= (x - y)(x + y)(x^2 + y^2)$
$= (x^2 - y^2)(x^2 + y^2)$
$= (x^2)^2 - (y^2)^2$
$= x^4 - y^4$

EXERCISE 4.4

Q1. Rationalize the denominator of the following.

(i) $\frac{3}{4\sqrt{3}}$

Solution:

$$= \frac{3}{4\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{4.3} = \frac{\sqrt{3}}{4}$$

(ii) $\frac{14}{\sqrt{98}}$

Solution:

$$= \frac{14}{\sqrt{98}} \cdot \frac{\sqrt{98}}{\sqrt{98}} = \frac{14\sqrt{98}}{98} = \frac{1}{14}\sqrt{49 \times 2} = \frac{1}{7}.7.\sqrt{2} = \sqrt{2}$$

(iii) $\frac{6}{\sqrt{8}\sqrt{27}}$

Solution:

$$= \frac{6}{\sqrt{8}\sqrt{27}} \cdot \frac{\sqrt{8}\sqrt{27}}{\sqrt{8}\sqrt{27}}$$
$$= \frac{6}{8 \times 27}.\sqrt{8}\sqrt{27}$$
$$= \frac{1}{36}.\sqrt{4.2}.\sqrt{9.3}$$
$$= \frac{1}{4 \times 9} \times 2 \times 3..\sqrt{2}\sqrt{3}$$
$$= \frac{1}{6}\sqrt{6} = \frac{\sqrt{6}}{6}$$

(iv) $\frac{1}{3+2\sqrt{5}}$

Solution:

$$= \frac{1}{3+2\sqrt{5}} \cdot \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$
$$= \frac{3-2\sqrt{5}}{(3)^2-(2\sqrt{5})^2} = \frac{3-2\sqrt{5}}{9-20}$$
$$= \frac{3-2\sqrt{5}}{-11}$$
$$= -\frac{1}{11}(3-2\sqrt{5})$$

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(v) $\dfrac{15}{\sqrt{31}-4}$

Solution:

$$= \frac{15}{\sqrt{31}-4} \times \frac{\sqrt{31}+4}{\sqrt{31}+4}$$

$$= \frac{15(\sqrt{31}+4)}{(3)^2-(4)^2} = \frac{15(\sqrt{31}+4)}{31-6}$$

$$= \frac{15(\sqrt{31}+4)}{15} = \sqrt{31}+4$$

(vi) $\dfrac{2}{\sqrt{5}-\sqrt{3}}$

Solution:

$$= \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= 2\,\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}^2-\sqrt{3}^2} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3} = \frac{2(\sqrt{5}+\sqrt{3})}{2} = \sqrt{5}+\sqrt{3}$$

(vii) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

Solution:

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2} = \frac{(\sqrt{3})^2-2\sqrt{3}+(1)^2}{3-1}$$

$$= \frac{3-2\sqrt{2}+1}{2} = \frac{4-2\sqrt{3}}{2}$$

$$= \frac{2(2-\sqrt{3})}{2} = 2-\sqrt{3}$$

(viii) $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

Solution:

$$= \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2)} = \frac{5+2\sqrt{15}+3}{5-3}$$

$$= \frac{8+2\sqrt{15}}{2} = \frac{2(4+\sqrt{15})}{2}$$

$$= 4+\sqrt{15}$$

Q2. Find the conjugate of $x+\sqrt{y}$.

(i)	$3+\sqrt{7}$	**(ii)**	$4-\sqrt{5}$	**(iii)**	$2+\sqrt{3}$
(iv)	$2+\sqrt{5}$	**(v)**	$5+\sqrt{7}$	**(vi)**	$4-\sqrt{15}$
(vii)	$7-\sqrt{}$[illegible]	[illegible]	[illegible] $\sqrt{2}$		

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Solution:

(i) Conjugate of $3 + \sqrt{7}$ is $3 - \sqrt{7}$.

(ii) Conjugate of $4 - \sqrt{5}$ is $4 + \sqrt{5}$.

(iii) Conjugate of $2 + \sqrt{3}$ is $2 - \sqrt{3}$.

(iv) Conjugate of $2 + \sqrt{5}$ is $2 - \sqrt{5}$.

(v) Conjugate of $5 + \sqrt{7}$ is $5 - \sqrt{7}$.

(vi) Conjugate of $4 - \sqrt{15}$ is $4 + \sqrt{15}$.

(vii) Conjugate of $7 \quad \sqrt{6}$ is $7 + \sqrt{6}$.

(viii) Conjugate of $7 + \sqrt{2}$ is $9 - \sqrt{2}$.

Q3. **(i)** **If $x = 2 - \sqrt{3}$, find $\frac{1}{x}$**

(ii) **If $x = 4 - \sqrt{7}$, find $\frac{1}{x}$**

(iii) **If $x = \sqrt{3} + 2$, find $x + \frac{1}{x}$**

Solution:

(i) $x = 2 - \sqrt{3}$

$$\frac{1}{x} = \frac{1}{2-\sqrt{3}}$$

$$= \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{2+\sqrt{3}}{(2)^2-(\sqrt{3})^2} = \frac{2+\sqrt{3}}{4-3}$$

$$= \frac{2+\sqrt{3}}{1} = 2 + \sqrt{3}$$

(ii) $x = 4 - \sqrt{17}$

$$\frac{1}{x} = \frac{1}{2-\sqrt{3}}$$

$$= \frac{1}{4-\sqrt{17}} \times \frac{4+\sqrt{17}}{4+\sqrt{17}}$$

$$= \frac{4+\sqrt{17}}{(4)^2-(\sqrt{17})^2} = \frac{4+\sqrt{17}}{16-17}$$

$$= \frac{4+\sqrt{17}}{-1} = -(4+\sqrt{17}) = -4 - \sqrt{17}$$

(iii) $x = \sqrt{3} + 2$

$$\frac{1}{x} = \frac{1}{\sqrt{3}+2}$$

$$= \frac{1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2} = \frac{\sqrt{3}-2}{(\sqrt{3})^2-(2)^2}$$

$$= \frac{\sqrt{3}-2}{4} = \frac{\sqrt{3}-2}{-1} = -(\sqrt{3}-2)$$

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$$= -\sqrt{3} + 2 = 2 - \sqrt{3}$$

$$x + \frac{1}{x} = \sqrt{3} + 2 + 2 - \sqrt{3} = 4$$

Q4. Simplify

(i) $\dfrac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \dfrac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

(ii) $\dfrac{1}{2+\sqrt{3}} + \dfrac{2}{\sqrt{5}-\sqrt{3}} + \dfrac{1}{2+\sqrt{5}}$

(iii) $\dfrac{2}{\sqrt{5}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{2}} - \dfrac{3}{\sqrt{5}+\sqrt{2}}$

Solution:

(i) $\dfrac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \dfrac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$$= \frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})+(1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$$

$$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{2\sqrt{5}-2\sqrt{6}}{5-3} = \frac{2(\sqrt{5}-\sqrt{6})}{2}$$

$$= \sqrt{5} - \sqrt{6}$$

(ii) $\dfrac{1}{2+\sqrt{3}} + \dfrac{2}{\sqrt{5}-\sqrt{3}} + \dfrac{1}{2+\sqrt{5}}$

$$= \frac{1}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} \cdot \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} + \frac{1}{2+\sqrt{5}} \cdot \frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$= \frac{2-\sqrt{3}}{4-3} + \frac{2(\sqrt{5}+\sqrt{3})}{5-3} + \frac{2-\sqrt{5}}{4-5}$$

$$= \frac{2-\sqrt{3}}{1} + \frac{2(\sqrt{5}+\sqrt{3})}{2} + \frac{2-\sqrt{5}}{-1}$$

$$= 2 - \sqrt{3} + \sqrt{5} + \sqrt{3} - 2 + \sqrt{5}$$

$$= 2\sqrt{5}$$

(iii) $\dfrac{2}{\sqrt{5}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{2}} - \dfrac{3}{\sqrt{5}+\sqrt{2}}$

$$= \frac{2}{\sqrt{5}+\sqrt{3}} \cdot \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} \cdot \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{2}{\sqrt{5}+\sqrt{2}} \cdot \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{2(\sqrt{5}-\sqrt{3})}{5-3} + \frac{(\sqrt{3}-\sqrt{2})}{1} - \frac{3(\sqrt{5}-\sqrt{2})}{5-2}$$

$$= \frac{2(\sqrt{5}-\sqrt{3})}{2} + \frac{(\sqrt{3}-\sqrt{2})}{1} - \frac{3(\sqrt{5}-\sqrt{2})}{3}$$

$$= \sqrt{5} - \sqrt{3} + \sqrt{3} - \sqrt{2} - (\sqrt{5} - \sqrt{2})$$

$$= \sqrt{5} - \sqrt{2} - \sqrt{5} + \sqrt{5} + \sqrt{2} = 0$$

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Q5. (i) If $x = 2 + \sqrt{3}$, find the value of $x - \frac{1}{x}$ and $\left(x - \frac{1}{x}\right)^2$

(ii) $x = \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} + \sqrt{2}}$, find the value of $x + \frac{1}{x}$, $x^2 + \frac{1}{x^2}$ and $x^3 + \frac{1}{x^3}$.

[Hint: $a^2 + b^2 = (a+b)^2 - 2ab$ and $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$.

Solution:

(i) $x = 2 + \sqrt{3}$

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1} = 2 - \sqrt{3}$$

$$x - \frac{1}{x} = 2 + \sqrt{3} - (2 - \sqrt{3})$$

$$= 2 + \sqrt{3} - 2 + \sqrt{3} = 2\sqrt{3}$$

and $\left(x - \frac{1}{x}\right)^2 = \left(2\sqrt{3}\right)^2 = 4 \times 3 = 12$

(ii) $x = \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} + \sqrt{2}}$

$$\frac{1}{x} = \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}}$$

$$x + \frac{1}{x} = \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} + \sqrt{2}} + \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}}$$

$$= \frac{(\sqrt{5} - \sqrt{2})^2 + (\sqrt{5} + \sqrt{2})^2}{(\sqrt{5})^2 + (2)^2}$$

$$= \frac{(\sqrt{5})^2 + (\sqrt{2})^2 + (\sqrt{5})^2 + (\sqrt{2})^2}{5-2}$$

$$= \frac{5+2+5+2}{3} = \frac{14}{3}$$

$$\left(x + \frac{1}{x}\right)^2 = \left(\frac{14}{3}\right)^2$$

$$x^2 + \frac{1}{x^2} + 2 = \frac{196}{9}$$

$$x^2 + \frac{1}{x^2} = \frac{196}{9} - 2$$

$$= \frac{196-18}{9} = \frac{178}{9}$$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

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$$= \left(\frac{14}{3}\right)^3 - 3\left(\frac{14}{3}\right)$$
$$= \frac{2744}{27} - \frac{14}{1}$$
$$= \frac{2744-378}{27} = \frac{2366}{27}$$

Q6. Determine the rational numbers a and b if

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1} = a + b\sqrt{3}$$

Solution:

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1} = a + b\sqrt{3}$$

$$\Rightarrow \quad \frac{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = a + b\sqrt{3}$$

$$\Rightarrow \quad \frac{3-2\sqrt{3}+1+3+2\sqrt{3}+1}{3-1} = a + b\sqrt{3}$$

$$\frac{8}{2} = a + b\sqrt{3}$$

or $\quad 4 = a + b\sqrt{3}$

$$4 + 0 = a + b\sqrt{3}$$

By comparing on both sides

$\Rightarrow \quad a = 4, \qquad b = 0$

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REVIEW EXERCISE 4

Q1. Multiple Choice Questions. Choose the correct answers.

(i) **$4x + 3y - 2$ is an algebraic.........**

(a) expression (b) sentence
(c) equation (d) in equation

(ii) **The degree of polynomial $4x^4 + 2x^2y$ is.........**

(a) 1 (b) 2 (c) 3 (d) 4

(iii) **$a^3 + b^3$ is equal to.........**

(a) $(a-b)(a^2 + ab + b^2)$
(b) $(a+b)(a^2 - ab + b^2)$
(c) $(a-b)(a^2 - ab + b^2)$
(d) $(a-b)(a^2 + ab - b^2)$

(iv) **$(3+\sqrt{2})(3+\sqrt{2})$ is equal to.........**

(a) 7 (b) -7 (c) -1 (d) 1

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(v) **Conjugate of surd $a+\sqrt{b}$ is.........**

(a) $-a+\sqrt{b}$ (b) $a-\sqrt{b}$

(c) $\sqrt{a}+\sqrt{b}$ (d) $\sqrt{a}-\sqrt{b}$

(vi) $\frac{1}{a-b}-\frac{1}{a+b}$ **is equal to ....**

(a) $\frac{2a}{a^2-b^2}$ (b) $\frac{2b}{a^2-b^2}$

(c) $\frac{-2a}{a^2-b^2}$ (d) $\frac{-2b}{a^2-b^2}$

(vii) $\frac{a^2-b^2}{a+b}$ **is equal to......**

(a) $(a-b)^2$ (b) $(a+b)^2$

(c) $a+b$ (d) $a-b$

(viii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$ **is equal to......**

(a) a^2+b^2 (b) a^2-b^2

(c) $a-b$ (d) $a+b$

Answers:

(i) a	**(ii) d**	**(iii) b**	**(iv) a**
(V) b	**(vi) b**	**(vii) b**	**(viii) c**

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Q2. Fill in the blanks.

(i) The degree of the polynomial $x^2y^2+3xy+y^3$ is.........

(ii) $x^2-4=$.........

(iii) $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)$(.........).

(iv) $2\left(a^2+b^2\right)=(a+b)^2+$ (.........)^2.

(v) $\left(x-\frac{1}{x}\right)^2=$ ....

(vi) Order of surd $\sqrt[3]{x}$ is ....

(vii) $\frac{1}{2-\sqrt{3}}=$ ....

Answers:

(i) **4** **(ii)** $(x+2)(x-2)$

(iii) $x^2-1+\frac{1}{x^2}$ **(iv)** $(a+b)^2(a-b)^2$

(v) $x^2-2+\frac{1}{x^2}$ **(vi)** **3**

(vii) $2+\sqrt{3}$

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Q3. If $x + \frac{1}{x} = 3$, find (i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$

Solution:

$$x + \frac{1}{x} = 3$$

(i) $$\left(x + \frac{1}{x}\right)^2 = (3)^2$$

$$x^2 + \frac{1}{x^2} + 2 = 9$$

$$x^2 + \frac{1}{x^2} = 9 - 2 = 7$$

(ii) $$\left(x^2 + \frac{1}{x^2}\right)^2 = (7)^2$$

$$x^4 + \frac{1}{x^4} + 2 = 49$$

$$x^4 + \frac{1}{x^4} = 49 - 2 = 47$$

Q4. If $x - \frac{1}{x} = 2$, find (i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$

Solution:

$$x - \frac{1}{x} = 2$$

(i) $$\left(x - \frac{1}{x}\right)^2 = (2)^2$$

$$x^2 + \frac{1}{x^2} - 2 = 4$$

(ii) $$\left(x^2 + \frac{1}{x^2}\right)^2 = (6)^2$$

$$x^4 + \frac{1}{x^4} + 2 = 36$$

$$x^4 + \frac{1}{x^4} = 36 - 2 = 34$$

Q5. Find the value of $x^3 + y^3$ and xy if $x + y = 5$ and $x - y = 3$

Solution:

$$x + y = 5, \quad x - y = 3$$

$$4xy = (x + y)^2 - (x - y)^2$$

$$= (5)^2 - (3)^2$$

$$4xy = 25 - 9 = 16$$

$$xy = 4$$

Now

$$x^3 + y^3 = (x + y)^3 - 3xy(x + y)$$

$$= (5)^3 - 3(4)(5) = 125 - 60 = 65$$

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Q6. **If $p = 2 + \sqrt{3}$, find**

(i) $p + \frac{1}{p}$ **(ii)** $p - \frac{1}{p}$

(iii) $p^2 + \frac{1}{p^2}$ **(iv)** $p^2 - \frac{1}{p^2}$

Solution:

$$p = 2 + \sqrt{3}$$

$$\frac{1}{p} = \frac{1}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{(2)^2-(3)^2} = \frac{2-\sqrt{3}}{4-3}$$

$$= \frac{2-\sqrt{3}}{1} = 2 - \sqrt{3}$$

(i) $p + \frac{1}{p} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

(ii) $p - \frac{1}{p} = 2 + \sqrt{3} - (2 - \sqrt{3})$

$$= 2 + \sqrt{3} - 2 + \sqrt{3} = 2\sqrt{3}$$

(iii) $p^2 + \frac{1}{p^2} = \left(p + \frac{1}{p}\right)^2 - 2$

$$= (4)^2 - 2$$

$$= 16 - 2 = 14$$

(iv) $p^2 - \frac{1}{p^2} = \left(p + \frac{1}{p}\right)\left(p - \frac{1}{p}\right)$

$$= (4)(2\sqrt{3})$$

$$= 8\sqrt{3}$$

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Q7. **If $q = \sqrt{5} + 2$, find**

(i) $q + \frac{1}{q}$ **(ii)** $q - \frac{1}{q}$

(iii) $q^2 + \frac{1}{q^2}$ **(iv)** $q^2 - \frac{1}{q^2}$

Solution:

$$q = \sqrt{5} + 2$$

$$\frac{1}{q} = \frac{1}{\sqrt{5}+2} \cdot \frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$= \frac{\sqrt{5}-2}{(\sqrt{5})^2-(2)^2} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5} - 2$$

(i) $q + \frac{1}{q} = \sqrt{5} + 2 + \sqrt{5} - 2 = 2\sqrt{5}$

(ii) $q - \frac{1}{q} = \sqrt{5} + 2 - (\sqrt{5} - 2)$

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$$= \sqrt{5} + 2 - \sqrt{5} + 2 = 4$$

(iii) $q^2 + \frac{1}{q^2} = \left(q + \frac{1}{q}\right)^2 - 2$

$$= (2\sqrt{5})^2 - 2$$

$$= 20 - 2 = 18$$

(iv) $q^2 - \frac{1}{q^2} = \left(q + \frac{1}{q}\right)\left(q - \frac{1}{q}\right)$

$$= (2\sqrt{5})(4)$$

$$= 8\sqrt{5}$$

Q8. Simplify

(i) $\frac{\sqrt{a^2+2}+\sqrt{a^2+2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}$ **(ii)** $\frac{1}{a-\sqrt{a^2-x^2}} - \frac{1}{a+\sqrt{a^2-x^2}}$

Solution:

(i) $\frac{\sqrt{a^2+2}+\sqrt{a^2+2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}$

$$= \frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}} \cdot \frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}+\sqrt{a^2-2}}$$

$$= \frac{(\sqrt{a^2+2}-2\sqrt{a^2+2}.\sqrt{a^2-2})+(\sqrt{a^2-2})^2}{(\sqrt{a^2+2})^2-(\sqrt{a^2-2})^2}$$

$$= \frac{a^2+2-2\sqrt{a^2-4}+a^2-2}{a^2+2-a^2+2}$$

$$= \frac{2a^2-2\sqrt{a^2-4}}{4}$$

$$= \frac{2(a^2-\sqrt{a^2-4})}{4}$$

$$= \frac{a^2-\sqrt{a^2-4}}{2}$$

(ii) $\frac{1}{a-\sqrt{a^2-x^2}} - \frac{1}{a+\sqrt{a^2-x^2}}$

$$= \frac{a+\sqrt{a^2-x^2}-(a-\sqrt{a^2-x^2})}{(a-\sqrt{a^2-x^2})(a+\sqrt{a^2-x^2})}$$

$$= \frac{a+\sqrt{a^2-x^2}-a+\sqrt{a^2-x^2}}{(a)^2-(\sqrt{a^2-x^2})^2}$$

$$= \frac{2\sqrt{a^2-x^2}}{a^2-(a^2-x^2)}$$

$$= \frac{2\sqrt{a^2-x^2}}{a^2-a^2+x^2} = \frac{2\sqrt{a^2-x^2}}{x^2}$$

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